



**NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY**

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science ; Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSOC; 07BSAM	LEVEL: 6
COURSE CODE: ODE602S	COURSE NAME: ORDINARY DIFFERENTIAL EQUATIONS
SESSION: NOVEMBER 2022	PAPER: THEORY
DURATION: 3 HOURS	MARKS: 80

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Prof A.S EEGUNJOBI
MODERATOR:	Prof S.A REJU

INSTRUCTIONS
<ol style="list-style-type: none">1. Answer ANY FOUR(4) questions in the booklet provided.2. Show clearly all the steps used in the calculations.3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

1. Solve the following initial value problems:

(a) $x^2y'(x) + 5x^3y(x) = e^{-x}$, $y(-1) = 0$, for $x < 0$ (5)

(b) $\sin xy'(x) + \cos xy(x) = 2e^x$, $y(1) = a$, $0 < x < \pi$ (5)

(c) If a constant number k of fish are harvested from a fishery per unit time, then a logistic model for the population $P(t)$ of the fishery at time t is given by

$$\frac{dP(t)}{dt} = -P(t)(P(t) - 5) - 4, \quad P(0) = P_0$$

i. Solve the IVP. (5)

ii. Determine the time when the fishery population becomes half of the initial population (5)

2. (a) If y_1 and y_2 are two solutions of second order homogeneous differential equation of the form

$$y''(x) + p(x)y'(x) + q(x)y(x) = f(x)$$

where $p(x)$ and $q(x)$ are continuous on an open interval I , derive the formula for $u(x)$ and $v(x)$ by using variation of parameters. (6)

(b) If

$$y_1(x) = x - \frac{1}{3}, \quad W(y_1, y_2) = -x^2 + \frac{2x}{3} - 1, \quad y_2(0) = 1$$

find $y_2(x)$ (7)

(c) Solve

$$8x^2y''(x) + 16xy'(x) + 2y(x) = 0 \quad (7)$$

3. (a) Find the general solution of

$$y^{iv}(x) + 2y''(x) + y = 0 \quad (6)$$

(b) Find the general solution of

$$y'''(x) - 6y''(x) + 11y'(x) - 6y = e^{-2x} + e^{-3x} \quad (7)$$

(c) Solve the following differential equations simultaneously

$$\frac{dx}{dt} + 5x(t) - 2y(t) = t, \quad \frac{dy}{dt} + 2x(t) + y(t) = 0 \quad (7)$$

4. (a) Calculate

$$\mathcal{L}\{9t^4 + 6t^{\frac{5}{2}}\} \quad (6)$$

(b) Using Convolution theorem, find

$$\mathcal{L}^{-1} \left\{ \frac{s^2}{(s^2 + 64)^2} \right\}$$

(7)

(c) Solve the following IVP:

$$2y''(t) - 6y'(t) + 4y(t) = 4e^{3t}, \quad y(0) = 5, \quad y'(0) = 7$$

(7)

5. (a) Find the radius of convergence of the following power series

$$\sum_{n=0}^{\infty} \frac{(3n)!}{(n!)^3} x^n$$

(5)

(b) Find the first five terms in the series solution of

$$y'(x) + y(x) + x^2 y(x) = \sin x, \quad \text{with } y(0) = a.$$

(5)

(c) Find series solution of IVP

$$5y''(x) + 10xy'(x) + 5(1 + x^2)y(x) = 0, \quad \text{with } y(0) = 3, \quad y'(0) = -1.$$

when the expansion is about the origin.

(10)

End of Exam!